

Engineering Mathematics – – intuitive and formal

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Working Group on Education and TP Technology
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1 Introduction to *ISAC*

2 Planned Extensions for STEM Education

Input to figural problem description

Worksheets with switching levels of abstraction

Extend program language with informal justifications

Conquer problems by managing sub-problems

3 Conclusions

Introduction to *ISAC*

- *ISAC* is designed for “pure” mathematics”:
 - 1 **check user input** automatically, **flexibly** and reliably:
Input establishes a *proof situation* (for *automated* proving) with respect to the logical context
 - 2 **give explanations** on request by learners:
All underlying mathematics knowledge is **transparent** due to the “LCF-paradigm” in Isabelle
 - 3 **propose a next step** if learners get stuck:
“next-step-guidance” due to Lucas-Interpretation.
- Engineers raised additional requirements for STEM (Science, Technology, Engineering and Mathematics)
- Answers to 4 requirements planned — and discussed here

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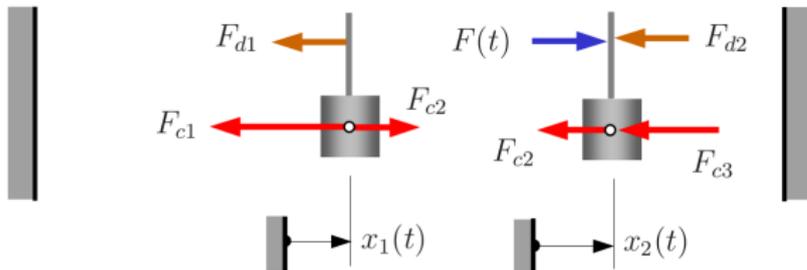
3 Conclusions

Figural input

Input of

- arrows at certain locations
- certain direction for arrows
- labels for arrows
- coordinate system with coordinates

to a figure like:



Automated check of input from formal specification

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Conclusions

. Problem [absorber, 2-mass-oscillator]

1 Specification:

2 Solution:

21 Problem [determine, 2-mass-oscillator, DiffEq]

$$22 \quad \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \ddot{x} + \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix} \dot{x} + \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_1 + c_2 \end{pmatrix} x = \begin{pmatrix} 0 \\ F \end{pmatrix}$$

23 Problem [solution, 2-mass-oscillator, homogen, DiffEq]

$$24 \quad x(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} (A_1 \cos \omega_1 t + B_1 \sin \omega_1 t) + \begin{pmatrix} 1 \\ -1 \end{pmatrix} (A_2 \cos \omega_2 t + B_2 \sin \omega_2 t),$$

25 Problem [particular, solution, 2-mass-oscillator, DiffEq]

$$26 \quad x_1(t) = \begin{pmatrix} 0 \\ a_1 \end{pmatrix} \sin \Omega t, \quad x_2(t) = \begin{pmatrix} 0 \\ a_2 \end{pmatrix} \sin \Omega t, \quad a_1 = \frac{F_0 c_2}{(c_1 + c_2 - m\Omega^2)^2 - c_2^2}, \quad a_2 = \frac{F_0 (c_1 + c_2 - m\Omega^2)}{(c_1 + c_2 - m\Omega^2)^2 - c_2^2}$$

27 Problem [complete, solution, 2-mass-oscillator, DiffEq]

$$28 \quad x(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} (A_1 \cos \omega_1 t + B_1 \sin \omega_1 t) + \begin{pmatrix} 1 \\ -1 \end{pmatrix} (A_2 \cos \omega_2 t + B_2 \sin \omega_2 t) + \begin{pmatrix} 0 \\ a_1 \end{pmatrix} \sin \Omega t,$$

$$a_1 = \frac{F_0 c_2}{(c_1 + c_2 - m\Omega^2)^2 - c_2^2}, \quad a_2 = \frac{F_0 (c_1 + c_2 - m\Omega^2)}{(c_1 + c_2 - m\Omega^2)^2 - c_2^2}$$

29 Problem [compute, spring]

2a $c_2 = 1.2345 \text{ N}$

. $c_2 = 1.2345 \text{ N}$

Numeric representation

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- . Problem [absorber, 2-mass-oscillator]
- 1 Specification:
- 2 Solution:
- 21 Problem [determine, 2-mass-oscillator, DiffEq]
- 22 $[2\ddot{x}_1 + 0.4\dot{x}_1 + 3.3x_1 - 0.22x_2 = 0, \quad 2\ddot{x}_2 + 0.4\dot{x}_2 - 0.22x_1 + 3.3x_2 = 0.6]$
- 23 Problem [solution, 2-mass-oscillator, homogen, DiffEq]
- 24 $[x_1(t) = 0.05e^{-0.1t}(\cos 0.81t + 3.85 \sin 0.81t),$
 $x_2(t) = 0.05e^{-0.1t}(\cos 0.81t + 3.85 \sin 0.81t)]$
- 25 Problem [particular, solution, 2-mass-oscillator, DiffEq]
- 26 $[x_1(t) = -0.05e^{-0.1t}0.59 \sin 1.69t, \quad x_2(t) = 0.05e^{-0.1t}0.59 \sin 1.69t]$
- 27 Problem [complete, solution, 2-mass-oscillator, DiffEq]
- 28 $[x_1(t) = 0.05e^{-0.1t}(\cos 0.81t + 3.85 \sin 0.81t - 0.59 \sin 1.69t),$
 $x_2(t) = 0.05e^{-0.1t}(\cos 0.81t + 3.85 \sin 0.81t + 0.59 \sin 1.69t)]$
- 29 Problem [compute, absorber]
- 2a $c_2 = 1.2345 \text{ N}$
- . $c_2 = 1.2345 \text{ N}$

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Extend programs with informal justifications

```

. partial_function diffeq_2_mass_oscil (m, l_0, [c_1, c_2],
                                     d, springs, dampers, sums) =
1   let
11  begin_parallel
1101    springs = Take springs "forces of springs"
111    parallel
1111    dampers = Take dampers "forces of dampers"
112    parallel
1121    sums = Take sums "mass times acceleration equals sum"
12  end_parallel
13  diffeq = Take sums ""
14  diffeq = Substitute [ springs, dampers ]
15  diffeq = Rewrite_Set normalise
16  diffeq = Rewrite_Set vectorify "switch to vector representation"
2   in
21  diffeq

```

More Expressive Worksheets

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21 Problem [determine, 2-mass-oscillator, DiffEq]:

211 Specification:

212 Solution:

2121

forces of springs

$$2122 \quad [F_{c1} = c_1 x_1, F_{c2} = c_2(x_2 - x_1), F_{c3} = c_1 x_2]$$

2123

forces of dampers

$$2124 \quad [F_{d1} = d\dot{x}_1, F_{d2} = d\dot{x}_2]$$

2125

mass times acceleration equals sum of all forces

$$2126 \quad [m\ddot{x}_1 = -F_{c1} + F_{c2} - F_{d1}, m\ddot{x}_2 = -F_{c2} - F_{c3} - F_{d2} + F]$$

2127

Substitute $[F_{c1}, F_{c2}, F_{c3}, F_{d1}, F_{d2}]$

$$2128 \quad [m\ddot{x}_1 = -c_1 x_1 + c_2(c_2 - x_1) - d\dot{x}_1, m\ddot{x}_2 = -c_2(c_2 - x_1) - c_1 x_2 - d\dot{x}_2 + F]$$

2129

Rewrite_Set normalise

$$212a \quad [m\ddot{x}_1 + d\dot{x}_1 + c_1 x_1 - c_2(x_2 - x_1) = 0, m\ddot{x}_2 + d\dot{x}_2 + c_2(x_2 - x_1) + c_1 x_1 = F]$$

212b

switch to vector representation

212c

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_1 + c_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ F \end{pmatrix}$$

22

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \ddot{x} + \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix} \dot{x} + \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_1 + c_2 \end{pmatrix} x = \begin{pmatrix} 0 \\ F \end{pmatrix}$$

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Conquer problems by managing sub-problems

General aspects in problem solving:

- Aspect 1: **Review** knowledge for relevance
- Aspect 2: **Select** relevant knowledge
- Aspect 3: **Arrange** subproblems to a sequence
- Aspect 4: Connect the **subproblems**
- Aspect 5: Accomplish **unit** conversions
- Aspect 6: **Solve** the subproblems

“Aspect” because not tackled sequentially.

Start Example

The screenshot shows a software window titled "Example browser" with a "Context On->Off" button. The left sidebar contains a tree view of examples, with "Etc" selected. The main content area displays a physics problem:

From a horizontally lying pipe with a diameter of 8 cm there are 5 liters of water flowing out per second. At what height is this pipe, if the horizontal distance between outlet and incidence on the floor is 80 cm?

Note: First determine the exit velocity (by use of the volume of water per second and of the cross-section area.)

Below the text is a diagram of a pipe of diameter d at a certain height. A blue parabolic arc represents the trajectory of water flowing out of the pipe and landing on a horizontal floor 80 cm away from the pipe's outlet.

Start Example

The screenshot shows a software window titled "Worksheet" with a menu bar containing "NEW", "Examples", "Theories", "Problems", and "Methods". To the right of the menu bar are buttons for "NEXT" and "AUTO". The main content area is split into two panes. The left pane is empty. The right pane contains the following text:

From a horizontally lying pipe with a diameter of 8 cm there are 5 liters of water flowing out per second. At what height is this pipe, if the horizontal distance between outlet and incidence on the floor is 80 cm?

Note: First determine the exit velocity (by use of the volume of water per second and of the cross-section area.)

Below the text is a diagram showing a horizontal pipe with diameter d on the left. A blue parabolic curve represents the trajectory of water falling from the pipe's outlet to a horizontal floor. The horizontal distance from the pipe's outlet to the point where the water hits the floor is labeled as 80 cm.

Modelling Phase finished

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The screenshot shows a software window with a menu bar (NEW, Examples, Theories, Problems, Methods) and buttons for NEXT and AUTO. The main area is divided into two panes. The left pane, titled 'Worksheet', contains the following text:

Model:
Given : Diameter $d = 8$ cm, FlowRate $\phi = 5$ l/s,
HorizontalDistance $s = 80$ cm
Where: $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : HeightOfPipe h

The right pane contains a text problem and a diagram. The text reads: "From a horizontally lying pipe with a diameter of 8 cm there are 5 liters of water flowing out per second. At what height is this pipe, if the horizontal distance between outlet and incidence on the floor is 80 cm? Note: First determine the exit velocity (by use of the volume of water per second and of the cross-section area.)" The diagram shows a horizontal pipe of diameter d on the left, with a curved blue line representing the trajectory of water falling to a horizontal floor at a distance of 80 cm.

Start Specification Phase

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The screenshot displays a software interface with two main windows. The top window, titled "Worksheet", contains the following text:

Model:
Given : Diameter $d = 8$ cm, FlowRate $\phi = 5$ l/s,
HorizontalDistance $s = 80$ cm
Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : $h_{\text{ghtOfPipe}} h$

The bottom window, titled "Problem browser", shows a hierarchical tree structure. The tree is expanded to show the following items:

- problem hierarchy
 - e_pblID
 - simplification
 - vereinfachen
 - probe
 - equation
 - univariate
 - LINEAR
 - root'
 - rational
 - polynomial
 - expanded
 - logarithmic
 - makeFunctionTo
 - diophantine
 - function
 - system
 - Biegelinien
 - Berechnung
 - test
 - tool
 - SignalProcessing
 - z_Transform

Start Specification Phase

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The screenshot displays two windows from a software application. The 'Worksheet' window contains the following text:

Model:
Given : Diameter $d = 8$ cm, FlowRate $\phi = 5$ l/s,
HorizontalDistance $s = 80$ cm
Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : HeightOfPipe h

Below the text is a large grey area with the text: d cm, ϕ l/s, s cm

At the bottom of the worksheet window, it says: Find: h m

The 'Problem browser' window shows a tree structure of methods:

- problem hierarchy
 - e_pblID
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 - vereinfachen
 - probe
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 - LINEAR
 - root'
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Aspect 1: relevant knowledge assumed to be present

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The screenshot displays a software interface with two main windows. The 'Worksheet' window on the left contains the following text:

Model:
 Given : Diameter $d = 8$ cm, FlowRate $\phi = 5$ l/s,
 HorizontalDistance $s = 80$ cm
 Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 Find : $h_{\text{rightOfPipe}}$ h

Below this text, the variables d cm, ϕ l/s, s cm are listed. At the bottom of the worksheet, the instruction 'Find: h m' is present.

The 'Problem browser' window on the right shows a hierarchical tree structure. The 'equation' folder is expanded, and the 'univariate' subfolder is highlighted. The 'solve (e_e, v_v)' problem is selected, and its details are shown in a table:

solve (e_e, v_v)	
Model:	
Given:	equality e_e solveFor v_v
Where:	e_e is_rateequation_in v_v
Find:	solutions v_v!
Relate:	

Aspect 2: select knowledge

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Worksheet

Model:

Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
HorizontalDistance $s = 80 \text{ cm}$

Where : $d > 0 \wedge \phi > 0 \wedge s > 0$

Find : $h_{\text{heightOfPipe}} h$

$d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$

Problem [rational, equation]

Find: $h \text{ m}$

Problem browser

Context On->Off Try Refine

- problem hierarchy
 - e_pblid
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solve (e_e, v_v)

Model:

Given:	equality e_e solveFor v_v
Where:	e_e is_rateequation_in v_v
Find:	solutions v_v!
Relate:	

Aspect 2: select relevant knowledge

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The screenshot displays a software interface with two main windows. The 'Worksheet' window on the left contains the following text:

Model:
Given : Diameter $d = 8$ cm, FlowRate $\phi = 5$ l/s,
HorizontalDistance $s = 80$ cm
Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : $h_{\text{ightOfPipe}} h$

Below this, the variables d cm, ϕ l/s, s cm are listed. Two problem boxes are shown:

- Problem [rational, equation]
- Problem [velocity-space-time, find-time]

The velocity equation $v = \frac{s}{t}$ is displayed below the second problem box. At the bottom of the worksheet, it says 'Find: h m'.

The 'Problem browser' window on the right shows a hierarchical tree structure:

- problem hierarchy
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Aspect 2: select relevant knowledge

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The screenshot shows a software interface with two main windows: "Worksheet" and "Problem browser".

Worksheet Window:

Model:
 Given : Diameter $d = 8$ cm, FlowRate $\phi = 5$ l/s,
 HorizontalDistance $s = 80$ cm
 Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 Find : $h_{\text{heightOfPipe}} h$

Problem browser Window:

- problem hierarchy
 - e_pblID
 - simplification
 - vereinfachen
 - probe
 - equation
 - univariate
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Worksheet Content:

d cm, ϕ l/s, s cm

Problem [rational, equation]

Problem [velocity-space-time, find-time]
 $v = \frac{s}{t}$

Problem [flow-rate, find-velocity]
 $v = \frac{\phi}{A_{\text{circle}}}$

Find: h m

Red arrows indicate the selection of relevant knowledge from the "Problem browser" to the "Worksheet".

Aspect 2: select relevant knowledge

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The screenshot shows a software interface with two main windows: "Worksheet" and "Problem browser".

Worksheet Window:

- Model:
Given : Diameter $d = 8$ cm, FlowRate $\phi = 5$ l/s,
HorizontalDistance $s = 80$ cm
Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : $h_{\text{heightOfPipe}} h$
- Variables: d cm, ϕ l/s, s cm
- Problem [rational, equation] (highlighted with a red arrow from the browser)
- Problem [velocity-space-time, find-time]
 $v = \frac{s}{t}$
- Problem [flow-rate, find-velocity]
 $v = \frac{\phi}{A_{\text{circle}}}$
- Problem [free-fall]
 $h = \frac{g}{2} \cdot t^2$
- Find: h m

Problem browser Window:

- Context On->Off Try Refine
- problem hierarchy
 - e_pblID
 - simplification
 - vereinfachen
 - probe
 - equation
 - univariate
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 - root'
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 - system
 - Biegelinien
 - Berechnung
 - test
 - tool
 - SignalProcessing
 - z_Transform

Red arrows indicate the selection of relevant knowledge from the problem browser to the worksheet. The "equation" category is selected, and a red arrow points to the "Problem [rational, equation]" box. Another red arrow points from the "equation" category to the "Problem [velocity-space-time, find-time]" box. A third red arrow points from the "equation" category to the "Problem [flow-rate, find-velocity]" box. A fourth red arrow points from the "equation" category to the "Problem [free-fall]" box.

Aspect 2: delete irrelevant knowledge

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The screenshot shows a software interface with two main windows. The 'Worksheet' window on the left contains the following text:

Model:
Given : Diameter $d = 8$ cm, FlowRate $\phi = 5$ l/s,
HorizontalDistance $s = 80$ cm
Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : $h_{\text{HeightOfPipe}} h$

Below this, the units 'd cm, ϕ l/s, s cm' are listed. There are four problem boxes, each with a title and a formula:

- Problem [rational equation] (crossed out with a red diagonal line)
- Problem [velocity-space-time, find-time]
$$v = \frac{s}{t}$$
- Problem [flow-rate, find-velocity]
$$v = \frac{\phi}{A_{\text{circle}}}$$
- Problem [free-fall]
$$h = \frac{g}{2} \cdot t^2$$

At the bottom of the worksheet, it says 'Find: h m'. The 'Problem browser' window on the right shows a tree structure of problem categories, including 'equation', 'function', 'system', 'Biegelminen', 'Berechnung', 'test', 'tool', and 'SignalProcessing'. The 'equation' category is expanded, showing sub-categories like 'univariate', 'LINEAR', 'root', 'rational', 'polynomial', 'expanded', 'logarithmic', 'makeFunctionTo', and 'diophantine'.

Aspect 2: select relevant knowledge

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The screenshot shows a software interface with two main windows. The left window, titled "Worksheet", contains a physics problem description and several problem boxes. The problem description includes: "Model: Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$, HorizontalDistance $s = 80 \text{ cm}$. Where : $d > 0 \wedge \phi > 0 \wedge s > 0$. Find : HeightOfPipe h ". Below this, there are four problem boxes: "Problem [area-of-circle]" with the formula $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$, "Problem [velocity-space-time, find-time]" with the formula $v = \frac{s}{t}$, "Problem [flow-rate, find-velocity]" with the formula $v = \frac{\phi}{A_{\text{circle}}}$, and "Problem [free-fall]" with the formula $h = \frac{g}{2} \cdot t^2$. At the bottom of the worksheet, it says "Find: h m". The right window, titled "Problem browser", shows a hierarchical tree of categories. An orange arrow points from the "equation" category in the tree to the "area-of-circle" problem box in the worksheet.

Worksheet Content:

Model:
Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
HorizontalDistance $s = 80 \text{ cm}$
Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : HeightOfPipe h

$d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$

Problem [area-of-circle]
 $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$

Problem [velocity-space-time, find-time]
 $v = \frac{s}{t}$

Problem [flow-rate, find-velocity]
 $v = \frac{\phi}{A_{\text{circle}}}$

Problem [free-fall]
 $h = \frac{g}{2} \cdot t^2$

Find: h m

Problem browser Content:

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Aspect 3: what is given, what has to be found?

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The screenshot shows a software interface with two main windows. The 'Worksheet' window on the left contains the following text:

Model:
Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
HorizontalDistance $s = 80 \text{ cm}$
Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : HeightOfPipe h

Below this, a diagram shows a vertical flow of subproblems:

- Top: $d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$
- Problem [area-of-circle]: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$
- Problem [velocity-space-time, find-time]: $v = \frac{s}{t}$
- Problem [flow-rate, find-velocity]: $v = \frac{\phi}{A_{\text{circle}}}$
- Problem [free-fall]: $h = \frac{g}{2} \cdot t^2$

At the bottom of the worksheet, it says 'Find: $h \text{ m}$ '.

The 'Problem browser' window on the right shows a tree structure of problem categories:

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Aspect 3: what is given, what has to be found?

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The screenshot shows a software interface with two main windows. The 'Worksheet' window on the left contains a physics problem and a diagram of its solution steps. The 'Problem browser' window on the right shows a hierarchical tree of problem categories.

Worksheet Content:

Model:
 Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
 HorizontalDistance $s = 80 \text{ cm}$
 Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 Find : HeightOfPipe h

Diagram of solution steps:

- Initial variables: $d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$
- Problem [area-of-circle]: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$ (units: $A \text{ cm}$)
- Problem [velocity-space-time, find-time]: $v = \frac{s}{t}$ (units: $v \text{ m/s}, t \text{ s}$)
- Problem [flow-rate, find-velocity]: $v = \frac{\phi}{A_{\text{circle}}}$ (units: $A \text{ m}, \phi \text{ m}^3/\text{s}, v \text{ m/s}$)
- Problem [free-fall]: $h = \frac{g}{2} \cdot t^2$ (units: $t \text{ s}, h \text{ m}$)

Final goal: Find: $h \text{ m}$

Problem browser Content:

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Aspect 3: try to connect “Given” and “Find”

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The screenshot displays a software interface with two main windows: a 'Worksheet' and a 'Problem browser'.

Worksheet Window:

- Model:**
 - Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$, HorizontalDistance $s = 80 \text{ cm}$
 - Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 - Find : $h_{\text{ightOfPipe}} h$
- Problem [area-of-circle]:**
 - Equation: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$
 - Units: $d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$
 - Variables: $A \text{ cm}^2$
- Problem [velocity-space-time, find-time]:**
 - Equation: $v = \frac{s}{t}$
 - Units: $v \text{ m/s}$
 - Variables: $t \text{ s}$
- Problem [flow-rate, find-velocity]:**
 - Equation: $v = \frac{\phi}{A_{\text{circle}}}$
 - Units: $A \text{ m}^2, \phi \text{ m}^3/\text{s}$
 - Variables: $v \text{ m/s}$
- Problem [free-fall]:**
 - Equation: $h = \frac{g}{2} \cdot t^2$
 - Units: $t \text{ s}, h \text{ m}$
- Find:** $h \text{ m}$

Problem browser Window:

- Context On->Off
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Aspect 3: try to connect "Given" and "Find"

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The screenshot shows a software interface with two main windows. The left window is titled "Worksheet" and contains a physics problem. The right window is titled "Problem browser" and shows a hierarchical tree of problem types.

Worksheet Content:

Model:
 Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
 HorizontalDistance $s = 80 \text{ cm}$
 Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 Find : HeightOfPipe h

The problem is decomposed into subproblems:

- Problem [area-of-circle]**: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$. This problem uses d (cm) and ϕ (l/s) as input and produces A (cm²) as output.
- Problem [velocity-space-time, find-time]**: $v = \frac{s}{t}$. This problem uses v (m/s) and s (m) as input and produces t (s) as output.
- Problem [flow-rate, find-velocity]**: $v = \frac{\phi}{A_{\text{circle}}}$. This problem uses A (m²) and ϕ (m³/s) as input and produces v (m/s) as output.
- Problem [free-fall]**: $h = \frac{g}{2} \cdot t^2$. This problem uses t (s) as input and produces h (m) as output.

The final goal is to find h (m).

Problem browser Content:

The problem browser shows a tree structure of problem types:

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The screenshot displays a software interface with two main windows: a 'Worksheet' and a 'Problem browser'.

Worksheet Window:

- Model:**
 - Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$, HorizontalDistance $s = 80 \text{ cm}$
 - Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 - Find : $h_{\text{HeightOfPipe}} h$
- Diagram:** A flowchart of subproblems. At the top, the variables $d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$ are listed. Below them are four problem boxes:
 - Problem [area-of-circle]:** $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$. It outputs $A \text{ cm}^2$.
 - Problem [velocity-space-time, find-time]:** $v = \frac{s}{t}$. It takes $v \text{ m/s}$ and $s \text{ m}$ as input and outputs $t \text{ s}$.
 - Problem [flow-rate, find-velocity]:** $v = \frac{\phi}{A_{\text{circle}}}$. It takes $A \text{ m}^2$ and $\phi \text{ m}^3/\text{s}$ as input and outputs $v \text{ m/s}$.
 - Problem [free-fall]:** $h = \frac{g}{2} \cdot t^2$. It takes $t \text{ s}$ as input and outputs $h \text{ m}$.
- Find:** $h \text{ m}$

Problem browser Window:

- Buttons: Context On->Off, Try Refine
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The screenshot displays a software interface with two main windows. The 'Worksheet' window on the left contains a physics problem and a flowchart of subproblems. The 'Problem browser' window on the right shows a hierarchical tree of problem types.

Worksheet Content:

Model:
 Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
 HorizontalDistance $s = 80 \text{ cm}$
 Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 Find : $h_{\text{HeightOfPipe}} h$

The flowchart in the worksheet shows the following subproblems and their relationships:

- Problem [area-of-circle]**: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$. It uses d (cm) and ϕ (l/s, s cm) as inputs and produces A (cm) as an output.
- Problem [velocity-space-time, find-time]**: $v = \frac{s}{t}$. It uses A (cm) and s (m) as inputs and produces t (s) as an output.
- Problem [flow-rate, find-velocity]**: $v = \frac{\phi}{A_{\text{circle}}}$. It uses ϕ (m³/s) and A (m) as inputs and produces v (m/s) as an output.
- Problem [free-fall]**: $h = \frac{g}{2} \cdot t^2$. It uses t (s) as an input and produces h (m) as an output.

The final goal is to find h (m). Red lines in the flowchart indicate the flow of information from the given values through the subproblems to the final goal.

Problem browser Content:

The 'Problem browser' window shows a tree structure of problem types:

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Aspect 3: try to connect “Given” and “Find”

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The screenshot displays a software interface with two main windows. The left window, titled "Worksheet", contains a physics problem model and a diagram of subproblems. The right window, titled "Problem browser", shows a hierarchical tree of mathematical and engineering topics.

Worksheet Content:

Model:
 Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
 HorizontalDistance $s = 80 \text{ cm}$
 Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 Find : HeightOfPipe h

The diagram in the worksheet shows the following subproblems and their relationships:

- Problem [area-of-circle]**: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$. It uses d (cm) and ϕ (l/s, s cm) as inputs and produces A (cm) as an output.
- Problem [velocity-space-time, find-time]**: $v = \frac{s}{t}$. It uses s (m) and v (m/s) as inputs and produces t (s) as an output.
- Problem [flow-rate, find-velocity]**: $v = \frac{\phi}{A_{\text{circle}}}$. It uses ϕ (m³/s) and A (m) as inputs and produces v (m/s) as an output.
- Problem [free-fall]**: $h = \frac{g}{2} \cdot t^2$. It uses t (s) as an input and produces h (m) as an output.

Red arrows indicate the flow of information: d and ϕ feed into the area-of-circle problem, which outputs A . A and ϕ feed into the flow-rate problem, which outputs v . v and s feed into the velocity-space-time problem, which outputs t . Finally, t feeds into the free-fall problem, which outputs h .

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Aspect 3: dangling connection

???

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Worksheet

Model:

Given : Diameter $d = 8\text{ cm}$, FlowRate $\phi = 5\text{ l/s}$,
HorizontalDistance $s = 80\text{ cm}$
Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : $h_{\text{HeightOfPipe}}\ h$

$d\text{ cm}, \phi\text{ l/s}, s\text{ cm}$

Problem [area-of-circle]
 $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$
 $A\text{ cm}^2$

Problem [velocity-space-time, find-time]
 $v = \frac{s}{t}$
 $v\text{ m/s}, t\text{ s}$

Problem [flow-rate, find-velocity]
 $v = \frac{\phi}{A_{\text{circle}}}$
 $v\text{ m/s}, \phi\text{ m}^3/\text{s}$

Problem [free-fall]
 $h = \frac{g}{2} \cdot t^2$
 $h\text{ m}, t\text{ s}$

Find: $h\text{ m}$

Problem browser

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Aspect 3: try another sequence

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Worksheet

Model:
 Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
 HorizontalDistance $s = 80 \text{ cm}$
 Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 Find : $h_{\text{HeightOfPipe}} h$

$d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$

Problem [area-of-circle]
 $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$
 $A \text{ cm}^2$

Problem [velocity-space-time, find-time]
 $v = \frac{s}{t}$
 $v \text{ m/s}, t \text{ s}$

Problem [flow-rate, find-velocity]
 $v = \frac{\phi}{A_{\text{circle}}}$
 $v \text{ m/s}, \phi \text{ m}^3/\text{s}$

Problem [free-fall]
 $h = \frac{g}{2} \cdot t^2$
 $h \text{ m}, t \text{ s}$

Find: $h \text{ m}$

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Aspect 3: flipped two subproblems

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The screenshot displays a software interface with two main windows: a 'Worksheet' and a 'Problem browser'.

Worksheet Window:

- Model:**
 - Given : Diameter $d = 8\text{ cm}$, FlowRate $\phi = 5\text{ l/s}$, HorizontalDistance $s = 80\text{ cm}$
 - Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 - Find : $h_{\text{HeightOfPipe}} h$
- Problem [area-of-circle]:**
 - Equation: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$
 - Variables: $d\text{ cm}$, $\phi\text{ l/s}$, $s\text{ cm}$
 - Intermediate result: $A\text{ cm}$
- Problem [flow-rate, find-velocity]:**
 - Equation: $v = \frac{\phi}{A_{\text{circle}}}$
 - Variables: $A\text{ cm}$, $\phi\text{ m}^3/\text{s}$
 - Intermediate result: $v\text{ m/s}$
- Problem [velocity-space-time, find-time]:**
 - Equation: $v = \frac{s}{t}$
 - Variables: $v\text{ m/s}$, $s\text{ m}$
 - Intermediate result: $t\text{ s}$
- Problem [free-fall]:**
 - Equation: $h = \frac{g}{2} \cdot t^2$
 - Variables: $t\text{ s}$
 - Final result: $h\text{ m}$
- Find:** $h\text{ m}$

Problem browser Window:

- Context: On->Off, Try Refine
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The screenshot displays a software interface with two main windows: a 'Worksheet' and a 'Problem browser'.

Worksheet Window:

- Model:**
 - Given : Diameter $d = 8\text{ cm}$, FlowRate $\phi = 5\text{ l/s}$, HorizontalDistance $s = 80\text{ cm}$
 - Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 - Find : HeightOfPipe h
- Diagram:** A vertical flow of four problem boxes connected by arrows, indicating a sequence of subproblems:
 - Problem [area-of-circle]:** $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$. An arrow labeled $d\text{ cm}, \phi\text{ l/s}, s\text{ cm}$ points to this box. Below it, an arrow labeled $A\text{ cm}$ points to the next box.
 - Problem [flow-rate, find-velocity]:** $v = \frac{\phi}{A_{\text{circle}}}$. An arrow labeled $\phi\text{ m}^3/\text{s}$ points to this box. Below it, an arrow labeled $v\text{ m/s}$ points to the next box.
 - Problem [velocity-space-time, find-time]:** $t = \frac{s}{v}$. An arrow labeled $s\text{ m}$ points to this box. Below it, an arrow labeled $t\text{ s}$ points to the next box.
 - Problem [free-fall]:** $h = \frac{g}{2} \cdot t^2$. An arrow labeled $t\text{ s}$ points to this box. Below it, an arrow labeled $h\text{ m}$ points to the final result.
- Final Result:** Find: $h\text{ m}$

Problem browser Window:

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The screenshot displays a software interface with two main windows: a 'Worksheet' and a 'Problem browser'.

Worksheet Window:

- Model:**
 - Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$, HorizontalDistance $s = 80 \text{ cm}$
 - Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 - Find : HeightOfPipe h
- Problem [area-of-circle]:**
 - Equation: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$
 - Variables: $A \text{ cm}^2$, $A \text{ m}^2$
- Problem [flow-rate, find-velocity]:**
 - Equation: $v = \frac{\phi}{A_{\text{circle}}}$
 - Variables: $v \text{ m/s}$
- Problem [velocity-space-time, find-time]:**
 - Equation: $v = \frac{s}{t}$
 - Variables: $t \text{ s}$
- Problem [free-fall]:**
 - Equation: $h = \frac{g}{2} \cdot t^2$
 - Variables: $h \text{ m}$
- Find:** $h \text{ m}$

Problem browser Window:

- Context: On->Off, Try Refine
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Worksheet

Model:
 Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
 HorizontalDistance $s = 80 \text{ cm}$
 Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 Find : HeightOfPipe h

$d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$

Problem [area-of-circle]
 $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$
 $A \text{ cm}^2$
 $\phi \text{ l/s}$
 $\phi \text{ m}^3/\text{s}$

Problem [flow-rate, find-velocity]
 $v = \frac{\phi}{A_{\text{circle}}}$
 $v \text{ m/s}$
 $s \text{ cm}$

Problem [velocity-space-time, find-time]
 $t = \frac{s}{v}$
 $t \text{ s}$

Problem [free-fall]
 $h = \frac{g}{2} \cdot t^2$
 $h \text{ m}$

Find: $h \text{ m}$

Problem browser

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The screenshot displays a software interface with two main windows: a 'Worksheet' and a 'Problem browser'.

Worksheet Window:

- Model:**
 - Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$, HorizontalDistance $s = 80 \text{ cm}$
 - Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 - Find : HeightOfPipe h
- Flowchart:**
 - Inputs: $d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$
 - Problem [area-of-circle]**: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$. Outputs: $A \text{ cm}^2, A \text{ m}^2$.
 - Problem [flow-rate, find-velocity]**: $v = \frac{\phi}{A_{\text{circle}}}$. Outputs: $v \text{ m/s}, \phi \text{ m}^3/\text{s}$.
 - Problem [velocity-space-time, find-time]**: $v = \frac{s}{t}$. Outputs: $s \text{ cm}, s \text{ m}$.
 - Problem [free-fall]**: $h = \frac{g}{2} \cdot t^2$. Output: $h \text{ m}$.
 - Find:** $h \text{ m}$

Problem browser Window:

- Context: On->Off, Try Refine
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 - makeFunctionTo
 - diophantine
 - function
 - system
 - Biegelinien
 - Berechnung
 - test
 - tool
 - SignalProcessing
 - z_Transform

Aspect 3: try to connect “Given” and “Find”

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Conclusions

The screenshot displays a software interface with two main windows: a 'Worksheet' and a 'Problem browser'.

Worksheet Window:

- Model:**
 - Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
 - HorizontalDistance $s = 80 \text{ cm}$
 - Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 - Find : HeightOfPipe h
- Diagram:** A flowchart showing the decomposition of the problem into subproblems:
 - Initial variables: $d \text{ cm}, \phi \text{ l/s}, s \text{ cm}$
 - Problem [area-of-circle]:** $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$. This problem outputs $A \text{ cm}$ and $A \text{ m}$.
 - Problem [flow-rate, find-velocity]:** $v = \frac{\phi}{A_{\text{circle}}}$. This problem outputs $v \text{ m/s}$ and $\phi \text{ m}^3/\text{s}$.
 - Problem [velocity-space-time, find-time]:** $v = \frac{s}{t}$. This problem outputs $t \text{ s}$.
 - Problem [free-fall]:** $h = \frac{g}{2} \cdot t^2$. This problem outputs $h \text{ m}$.
- Find:** $h \text{ m}$

- Problem browser Window:** A tree view showing a hierarchy of mathematical and engineering concepts:
- problem hierarchy
 - e_pblID
 - simplification
 - vereinfachen
 - probe
 - equation
 - univariate
 - LINEAR
 - root'
 - rational
 - polynomial
 - expanded
 - logarithmic
 - makeFunctionTo
 - diophantine
 - function
 - system
 - Biegelinien
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 - tool
 - SignalProcessing
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Aspect 3: all connections finished

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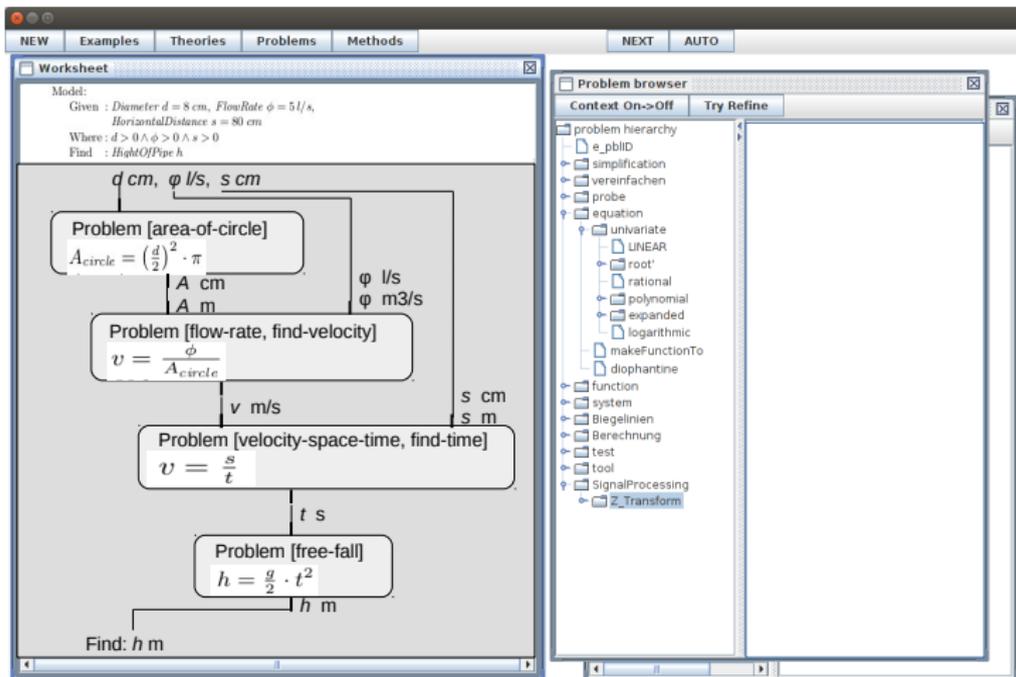
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Aspect 4: care about unit conversions

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Conclusions

The screenshot shows a software interface with two main windows: a 'Worksheet' and a 'Problem browser'.

Worksheet Window:

- Model:
 - Given : Diameter $d = 8\text{ cm}$, FlowRate $\phi = 5\text{ l/s}$, HorizontalDistance $s = 80\text{ cm}$
 - Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
 - Find : HeightOfPipe h
- Diagram showing a sequence of subproblems:
 - Top: $d\text{ cm}, \phi\text{ l/s}, s\text{ cm}$
 - Problem [area-of-circle]: $A_{\text{circle}} = \left(\frac{d}{2}\right)^2 \cdot \pi$. Units: $A\text{ cm}$, $A\text{ m}$.
 - Problem [flow-rate, find-velocity]: $v = \frac{\phi}{A_{\text{circle}}}$. Units: $\phi\text{ l/s}$, $\phi\text{ m}^3/\text{s}$.
 - Problem [velocity-space-time, find-time]: $v = \frac{s}{t}$. Units: $v\text{ m/s}$, $s\text{ cm}$, $s\text{ m}$.
 - Problem [free-fall]: $h = \frac{g}{2} \cdot t^2$. Unit: $t\text{ s}$.
 - Final result: Find: $h\text{ m}$.

Problem browser Window:

- Context On->Off Try Refine
- problem hierarchy
 - e_pblID
 - simplification
 - vereinfachen
 - probe
 - equation
 - univariate
 - LINEAR
 - root'
 - rational
 - polynomial
 - expanded
 - logarithmic
 - makeFunctionTo
 - diophantine
 - function
 - system
 - Biegelinien
 - Berechnung
 - test
 - tool
 - SignalProcessing
 - z_Transform

Transition to Solving Phase: units only

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Conclusions

The screenshot displays a software interface with two main windows. The left window, titled "Worksheet", contains a physics problem and its solution. The problem is: "Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$, HorizontalDistance $s = 80 \text{ cm}$. Where : $d > 0 \wedge \phi > 0 \wedge s > 0$. Find : $h_{\text{heightOfPipe}} h$ ". The solution is presented in several steps: 1. "Problem [area-of-circle]: $A_{\text{circle}} \text{ cm}$ Unit_conversion $\text{cm}^2_m^2$ ", 2. $A_{\text{circle}} \text{ m}$, 3. $\phi = 5 \frac{\text{l}}{\text{s}}$ Unit_conversion l_m^3 , 4. $\phi = 0,005 \frac{\text{m}^3}{\text{s}}$, 5. "Problem [flow-rate, find-velocity]: $v \frac{\text{m}}{\text{s}}$ ", 6. $s = 80 \text{ cm}$ Unit_conversion cm_m , 7. $s = 0,8 \text{ m}$, 8. "Problem [velocity-space-time, find-time]: $t \frac{\text{m}}{\text{s}}$ ", 9. "Problem [free-fall]: $h \text{ m}$ ". The right window, titled "Problem browser", shows a hierarchical tree of mathematical and physics concepts, including "equation", "function", "system", "Biegelinien", "Berechnung", "test", "tool", and "SignalProcessing". The "SignalProcessing" folder is expanded, showing a sub-entry "z_Transform".

Solving Phase finished with complete calculation

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The screenshot displays a software interface with two main windows. The 'Worksheet' window on the left contains the following text:

Model:
Given : Diameter $d = 8 \text{ cm}$, FlowRate $\phi = 5 \text{ l/s}$,
HorizontalDistance $s = 80 \text{ cm}$
Where : $d > 0 \wedge \phi > 0 \wedge s > 0$
Find : $h_{\text{heightOfPipe}} h$

Solution:

Problem [area-of-circle]
 $A_{\text{circle}} = 50 \text{ cm}^2$ Unit_conversion $\text{cm}^2_m^2$
 $A_{\text{circle}} = 0,005 \text{ m}^2$ Take.given ϕ
 $\phi = 5 \frac{\text{l}}{\text{s}}$ Unit_conversion l_m^3
 $\phi = 0,005 \frac{\text{m}^3}{\text{s}}$

Problem [flow-rate, find-velocity]
 $v = 1 \frac{\text{m}}{\text{s}}$
 $s = 80 \text{ cm}$ Unit_conversion cm_m
 $s = 0,8 \text{ m}$

Problem [velocity-space-time, find-time]
 $t = 0,8 \frac{\text{m}}{\text{s}}$

Problem [free-fall]:
 $h = 3,2 \text{ m}$ Check_postcond [composed, movement, no-6]

The 'Problem browser' window on the right shows a hierarchical tree structure of mathematical concepts. The tree includes:

- problem hierarchy
 - e_pblID
 - simplification
 - vereinfachen
 - probe
 - equation
 - univariate
 - LINEAR
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 - expanded
 - logarithmic
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 - diophantine
 - function
 - system
 - Biegelinien
 - Berechnung
 - test
 - tool
 - SignalProcessing
 - z_Transform**

Conclusions

- TP-technology is a powerful base for educational SW
 - student can rely on “yes” | “no” from system
 - powerful type systems
 - is still open source
 - quickly growing body of mechanised math
 - “self-referentiality” can foster abstraction
- *ISAC* can be extended for STEM with reasonable effort
- ... as soon as respective knowledge is mechanised:
! promotion in cooperation with FM !
- Hope for co-workers and analogous projects
(in addition to R.J.Back)

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Thank you for Attention!